REVIEW

by Prof. Dimcho Kostov Stankov, PhD

Concerning: Materials submitted for participation in a competition for academic position
"Professor" at Bishop Konstantin Preslavsky University of Shumen
in Higher Education Area: 4. Natural sciences, mathematics and informatics
Professional Field: 4.5 Mathematics (Mathematical Analysis)

1. General presentation

For participation in the announced competition for professor only one candidate Assoc. Prof., D. Sc. Sevdzhan Ahmedov Hakkaev, from the Department of Mathematical Analysis at the Faculty of Mathematics and Informatics of Bishop Konstantin Preslavsky University of Shumen has submitted documents. The documents presented by him are in accordance with the Rules for the development of the academic staff of Shumen University.

In order to participate in the competition, the applicant offered one textbook and 12 scientific papers published in scientific journals, which do not repeat the ones submitted for the degrees of Doctor, Doctor of Science and the academic position of Associate Professor. Assoc. Prof. Sevdzhan Hakkaev has a total of over 40 scientific papers, one textbook and two dissertations for the degreesDoctor and Doctor of Mathematical Sciences.

II. Educational and pedagogical activity

Assoc. Prof. S. Hakkaev graduated with a major in Mathematics, a Master's Degree the University of Shumen in 1995

Teaching at the University of Shumen.

Assoc. Prof. S. Hakkaev began his teaching career at the Shumen University in 1995. Since 2004 he is a doctor of Mathematics, since 2007 he is an Associate professor, and since 2010 - Doctor of mathematical sciences. He lectures on: Mathematical Analysis I. II, III and IV, Complex Analysis, Ordinary Differential Equations, Functional Analysis, General Topology. Assoc. Prof. S. Hakkaev is the author of academic programs for the specialties: Mathematics, Mathematics and Informatics, Informatics, Economics, Physics and Astronomy. Students in these and other specialties use the teaching aids of S.Hakkaev.

Teaching at the Mathematics Department at Yeditepe University, Istanbul, Turkey - lectures on Differential Equations and Calculus for Economists. (February 2012 - June 2012).

Teaching at the Mathematics Department at the University of Kansas, USA – lectures On Vector Calculus and Trigonometry (August 2010 - December 2010).

Teaching at Zhejiang Normal University, China - lectures on Ill - posedness of integrable evolution equations, April - May 2010.

Teaching at Aidan University in Istanbul.

Assoc. Prof. S. Hakkaev supervised two PhD students in Mathematical Analysis – Turhan Suleimanov and Mehmed Kodja.

III. Research activity.

The scientific interests of Assoc. Prof. S. Hakkaev are related to:

1. Stability to periodic and solitary waves.
2. Nonlinear dispersive equations.
3. Spectral inverse problem.

The articles presented are co-authored with mathematicians from Bulgaria, Brazil and USA. The numbering below is from the list of publications submitted by the author.

In the article [2] the authors consider the system introduced by Benny, which models the interaction between short and long waves:

$$\begin{cases} iu_t + u_{xx} = uv + \beta |u|^2 u \\ v_t = \left(|u|^2 \right)_x \\ u(x,0) = u_0(x), v(x,0) = v_0(x) \end{cases} \qquad (1)$$

where $u(x,t)$ is a complex – valued function representing the short wave and $v(x,t)$ is a real – valued function representing the long wave. The Cauchy theorem for the problem (1) is proved and conditions are found under which Cauchy problem is not locally wellposed.

In [3] is considered a family of non-evolutionary partial differential equations known as the Holm–Staley b-family which includes theintegrableCamasa – Holm and Degasperis - Procesi equations

$$m_t + um_x + bu_x m = 0,$$

with $m = u - u_{xx}, u(x,t)$, representing the fluid velocity, while the constant b is a balance or a bifurcation parameter for thesolution behavior. The authors show that the solution map is not uniformly continuous. The proof relies on a construction of smooth periodic traveling waves with small amplitude.

The existence and stability of spatially periodic waves $\left(e^{i\omega t}\varphi_\omega, \psi_\omega\right)$ in the Klein – Gordon – Zakharov (KGZ) system:

$$\begin{cases} u_{tt} - u_{xx} + u + uv = 0 \\ v_{tt} - c^2 v_{xx} = \left(|u|^2\right)_{xx} \end{cases}$$

is studied in [5]. It is hown a local existence result for low regularity initial data. Then theauthors construct a one-parameter family ofperiodic dnoidal waves for (KGZ) system when the period is bigger than $\sqrt{2}.\pi$. These waves are stable whenever an appropriate function satisfies the standard Grillakis–Shatah–Strauss type condition.

The object of study in [9] is the Klein-Gordon equation in 1 + 1 dimensions

$$u_{tt} - u_{xx} + u + |u|^{p-1} u = 0.$$

In particular, of interest is the spectral stability / instability (with respect to perturbations of the same period) of traveling - standing periodic solitons:

$$u(t,x) = e^{i\omega t}.e^{iq(x-ct)}\varphi_{\omega,c}(x - ct)$$

which are of cnoidal (p = 2), dnoidal (p = 3) or more general type (p =5). The results in this article generalize recent work on the simpler case of standing waves of Natali – Pastor and Natali - Cardoso.

In [11] the authors construct various periodic travelling waves solutions of the Ostrovsky /Hunter - Saxton/ short pulse equation:

$$\left(u_t + \left(f(u)\right)_x\right)_x = u$$

and its KdV regularized version. The stability of a class of periodic waves is investigated. For the short pulse model is constructed a family of travelling peakons with corner crests. It is shown that the peakons are spectrally stable as well.

The short pulse models in a symmetric spatial interval, subject to periodic boundary conditions are considered in [12]:

$$\left(u_t + \left(f(u)\right)_x\right)_x = u \quad . \qquad (3)$$

The main interest of the authors is the stability of explicit and classical traveling waves for the short pulse equation (3) of the form $u(t,x) = \varphi(x - ct)$. It is shown that the waves are spectrally stable for all wave speeds $c > 0$.

In [8], the linear stability of periodic waves for the Boussinesq equation

$$u_{tt} + u_{xxxx} - u_{xx} + \left(f(u)\right)_{xx} = 0,$$

where $f(p) = u^p$ ($p = 2,3$) and for the Klein-Gordon-Zakharov system:

$$\begin{cases} u_{tt} - u_{xx} + u + uv = 0 \\ v_{tt} - v_{xx} = \dfrac{1}{2}\left(|u|^2\right)_{xx} \end{cases}$$

is investigated. For a wide class of solutions, the authors completely and explicitly characterize their linear stability (instability) when the perturbations are taken with the same period as the waves.

The object of study in [7] is the spectral stability of the traveling waves for the Boussinesq system:

$$\begin{cases} \eta_t + u_x + \left(\eta u\right)_x + au_{xxx} - b\eta_{xxt} = 0 \\ u_t + \eta_x + uu_x + c\eta_{xxx} - du_{xxt} = 0 \end{cases}$$

The authors provide a complete rigorous characterization of the spectral stability in all cases for which $a = c < 0, b > 0$.

In [6], the nonlinear Schrödinger equation

$$iu_t + u_{xx} + |u|^p u = 0,$$

is considered. In these cases $p = 1$ and $p = 2$ it is shown that the corresponding periodic waves are orbiting stable. In [10], the authors study periodic standing waves for the Schrödinger system

$$\begin{cases} iu_t + u_{xx} + \left(\beta |u|^4 + 2\sigma |u|^2 |v|^2 + \sigma |u|^4 \right) u = 0 \\ iv_t + v_{xx} + \left(\sigma |u|^4 + 2\sigma |u|^2 |v|^2 + \gamma |v|^4 \right) v = 0 \end{cases},$$

where u and v are complex - significant functions and α, σ, γ are real parameters.

The results obtained by Assoc. Prof. S. Hakkaev are of high scientific value. All of the competition articles have been published in high Impact factor journals (1.245, 1.452, 1.640, 1.677, 1.829). The citations that do not repeat the ones submitted for the degrees of Doctor, Doctor of Science and the academic position of Associate Professorare 87 in number.

Associate professor Hakkaev has participated in the following scientific and educational projects:

1. At the University level: projects during 2009, 2010, 2015, 2016, 2018, 2019.

2. At a national level:

He has been a manager of a national project, DDVU 02/91/22.12.2010, titled: "Non-linear Equations and spectral theory of operators", awarded by the fund "Scientific investigations", Incentives for scientific research in the National Universities.

3. Postdoctoral project: FAPESP/Brazil Research Grant (August 2007 - July 2008), Institute for Mathematics and Statistics, University of San Paulo, Brasil.

The applicant has published reviews in AMS Mathematical Reviews, and article reviews for the following journals:

Journal of Differential Equations
- Journal of Mathematical Analysis and Applications
- Nonlinear Analysis, TMA
- Physics Letters A.
- Discrete and Continuous Dynamical Systems-Series A
- Nonlinear Analysis, RWA
- Applicable Analysis
- Mathematical Modeling and Analysis
- Abstract and Applied Analysis
- Journal of Inequalities and Applications
- Mathematical Biosciences
- Computer Physics Communications.
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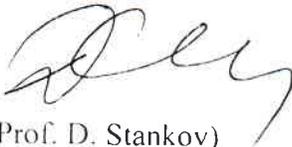## IV. Personal impressions and recommendations.

I know Assoc. Prof. S. Hakkaevas an established professional in his field. I have no criticisms of the scientific production submitted for the competition.

CONCLUSION

The submitted documents and materials fully comply with the requirements of the Law on the Development of the Academic Staff of the Republic of Bulgaria (LDASRB), the Rules for the implementation of the LDASRB and the Regulations for the Development of the Academic Staff of the Shumen University. The minimum required points from the scientific metric indicators for the academic position "Professor" are repeatedly exceeded. Based on the results of the research and teaching activity, I give my positive assessment and recommend to the Scientific Jury to select Assoc. Prof. SevdzhanHakkaev in the academic position of "Professor" at Bishop Konstantin Preslavsky University of Shumen in Higher Education Area: 4. Natural sciences, mathematics and informatics, Professional field: 4.5 Mathematics (Mathematical Analysis).

Shumen
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Prepared the review:

(Prof. D. Stankov)