

## **REVIEW REPORT**

**on the competition for the occupation of the academic position PROFESSOR  
in the field of higher education 4. Natural sciences, mathematics and  
informatics, professional field 4.5 Mathematics (Computational Mathematics),  
Published in the State Gazette, No 55 of 12.07.2019 for the needs of Konstantin  
Preslavsky University of Shumen, Faculty of Mathematics and Informatics**

**An only candidate: Assoc. Prof. Vejdi Ismailov Hasanov**

**Reviewer: Prof. Andrey Borisov Andreev, DSc**

### **1. Short details of the applicant**

The participant in the competition for the position of professor, Vejdi Ismailov Hasanov graduated as a “Master in mathematics” in 1997 and a “mathematics teacher” at the Faculty of Mathematics and Informatics (FMI) of the Bishop Konstantin Preslavski University of Shumen (SHU). In 1997 he has started his career as an assistant at FMI of SHU where he has been working ever since. He goes through all academic titles from assistant to associate professor.

In 2003 Vejdi Hasanov successfully defended his doctoral thesis for obtaining the PhD degree on subject “Solutions and perturbation theory of nonlinear matrix equations”. In 2007 he received the position of Associate Professor in Computational Mathematics at SHU.

Associate Professor Hasanov's scientific and pedagogical work is determined by his lasting and purposeful interest in the theory and applications of the numerical methods. These scientific interests have found a favorable environment in the staff of the University of Shumen. In this line of thought, the role of Prof. Milko Petkov as a teacher and scientist with a significant contribution to SHU and the development of mathematical research in it should be emphasized.

Associate Professor Hasanov has impressive administrative and management activities at SHU. From 2008 to 2012 he was the Deputy Dean for educational activities and from 2012 to 2016 - the Deputy Dean for Research and Administrative Activities of the Faculty of Mathematics and Informatics. From September 2016 to January 2018 he was the Head of the Department of Economics and Mathematical Modeling. From 2017 until now

Associate Professor Hasanov is the Dean of the Faculty of Mathematics and Informatics. During the accreditation procedures at FMI I have personal impressions of his good qualities and practices as a supervisor and organizer.

## **2. General description of the materials presented**

As an application for the competition I received the following materials, prepared by Assoc. Prof. Hasanov:

- a document folder containing a professional CV; higher education diploma; diploma for PhD degree; certificate of scientific title "Associate Professor"; a list of publications with which the candidate participates in the competition; reference to citations; references for additional indicators, as well as for fulfillment of the minimum national requirements under Art. 26, para. 2 and Art. 26, para. 3 of ZRASRB; a statement of authorship and abstracts of the textbooks, teaching aids and publications for the competition;
- copies of scientific publications in accordance with the list provided;
- two textbooks and one textbook manual.

The applicant participates in the competition with 12 scientific publications. All of them are reviewed and published in journals or proceedings of international conferences. Six of the articles have an impact factor which convincingly signifies the quality of scientific output.

Five of the articles presented are independent and the rest are co-authored. All publications are in English.

There are also three teaching aids attached to the competition materials. They are intended for students and doctoral students at the Faculty of Mathematics and Informatics of SHU.

The competition materials contain information about the candidate's work on scientific projects. For seven years he has been a member of the University Committee for Organization, Conducting, Reporting and Ranking of Projects under Ordinance No. 9 of the MES.

Assoc. Prof. Hasanov was the coordinator of a national project funded by the EU, as well as the supervisor of six projects at university level. He is currently working on two more national projects related to information and communication technologies.

The candidate for the "Professor" competition successfully supervises two PhD students who are in the final stages of preparing and defending their dissertation. He has

also completed three scientific mobility activities at Aydın University - Istanbul and Tekirda University in the Republic of Turkey.

### 3. Scientific contributions in the publications presented

The articles proposed for review are devoted to the approximation and numerical solution of nonlinear matrix equations or nonlinear functional equations. Matrix equations are an important part of the control theory for solving economic and financial problems, as well as for systems of ordinary differential equations.

I will focus on the more important scientific results obtained by Assoc. Prof. Hasanov, dividing them thematically.

#### 3.1. Perturbation estimates in the matrix equations

Articles [1], [4], [8], [9], [10] and [12] are devoted to this topic.

In [1] have been obtained the perturbation estimates of the two equations

$$X \pm A^* X^{-1} A = Q,$$

where  $A$  and  $Q$  are given matrices and  $A^*$  is the conjugate transpose of  $A$ . Two perturbation estimates are obtained for the maximal positive definite solutions for the considered matrix equations. These estimates are of a more general nature than those previously available.

The quality of iterative methods for matrix equations depends on the way of perturbation of the given equation and the choice of the initial approximation. Therefore, obtaining perturbation estimates is important in practical terms for numerical implementation.

In [4], the results of the perturbation theory for the algebraic Riccati equation are given in the presence of an additional linear operator of the unknown matrix. Perturbation estimates for stability to this operator have been proven.

The unique positive definite solution to a matrix equation is called extreme. In [8] has been obtained perturbation estimates for such an equation solution

$$X - \sum_{i=1}^m A_i^* X^{-1} A_i = Q. \quad (*)$$

The main result (Theorem 2) also raises the question of how to choose an appropriate positive definite matrix that provides the obtained estimates.

Similar results as in [8] are obtained in [9] but for equation

$$X + \sum_{i=1}^m A_i^* X^{-1} A_i = Q. \quad (**)$$

Here, the perturbation equation has matrix coefficients slightly different from those of (\*\*).

In [10], the results of [8] have been continued by discussing the choice of the matrix  $P$  and some interesting numerical experiments have been presented.

The work [12] is somewhat of an overview, discussing the perturbation estimates for the equation (\*) obtained by other authors. Interesting numerical experiments are tested here and on this basis a comparison is made with the results of an article [8].

### 3.2. Existence of a positive definite solution

In articles [3] and [11] have been thoroughly considered the equation

$$X + A^* X^{-1} A - B^* X^{-1} B = I, \quad (1)$$

where  $A$  and  $B$  are given matrices, and  $I$  is the identity matrix.

In [3], the conditions under which (1) has a unique positive definite solution are given (Theorem 5). In [11], the necessary and sufficient conditions for existence Hermitian positive definite solutions are proven.

These two articles discuss iterative procedures for finding positive definite solutions. Numerous experiments are also presented to support the proposed algorithms.

I believe that the results contained in paper [11] are some of the most comprehensive and in-depth results presented in the competition.

### 3.3. Iterative methods for solving nonlinear equations

Article [2] analyzes the classical combined Newton and secant method, as well as one of its modification to solve the nonlinear operator equation  $F(x)=0$ . Local convergence is proved and numerical examples are discussed.

In [5], the convergence rate is proved for the iterative Newton's method for finding the largest positive definite solution of the equation (\*\*). It is proved under what conditions this method is linear and when there is a quadratic rate of convergence. This result is expanded in an article [6], where, in addition to the mentioned method, much more iterative procedures applied to equation (\*\*) are considered.

Article [7] examines three iterative methods applied to equation (1). As in [6], the convergence to the maximum positive definite solution is proved.

#### **4. Educational and pedagogical activity**

It is clear from the report that Associate Professor Vejdi Hasanov has approximately 22 years of teaching experience at SHU. As an assistant he leads seminars and laboratory exercises in numerical methods and mathematical optimization, and after his habilitation is a holder in these disciplines. I am well aware of his reputation of being a teacher who is well versed in the subjects he teaches.

The application to the contest contains three textbooks with the sole author Vejdi Hasanov. They are intended for students and doctoral students at SHU.

One of them is "Lectures on Numerical Methods". It has an e-book version. The classical numerical methods of algebra and analysis are outlined and explained. A good impression is that after each paragraph there are questions and tasks to the material.

The second textbook is "Linear Optimization". The discipline of the same name is studied not only by students in mathematics, but also by those in economics.

"Guide to Numerical Methods with Matlab" is a teaching tool that is needed first and foremost for students' lab work. The usefulness of this tutorial is enhanced by the fact that a reference book for built-in functions and procedures for solving problems on the numerical methods studied is provided.

Associate Professor Hasanov successfully guided 10 students to receive Bachelor's Degree and / or Master's Degree. He is the scientific advisor of two PhD students, who I know are working very well and are about to finish their dissertation shortly.

#### **5. Citations**

In this respect, the data of Associate Professor Vejdi Hasanov is remarkable. The articles in which he is the author or co-author have been cited about 350 times. The most frequent citations are in specialized international journals and in magazines with impact factor and / or impact rank.

#### **6. Critical notes and recommendations**

I have no significant critical comments on the research and teaching activity of Associate Professor Hasanov. I just want to point out that I did not receive information about:

- the specific requirements of the University of Shumen for the occupation of the academic position "Professor";

- a course (s) with the relevant coursework for which the competition has been determined;

- a decision of the FB of the FMI to announce the competition for professorship (copy of protocol).

Here is the place to recommend Vejdi Hasanov to write a monograph on iterative methods for solving matrix equations. For example, on a personal level, I am interested in cases where the matrices are ill conditioned or when they are of a very high order.

### **7. Personal impressions of the applicant**

I have known Vejdi Ismailov Hasanov for more than 10 years. I have chaired three times an expert group for assessment and accreditation at the Faculty of Mathematics and Informatics of the University of Shumen.

I can responsibly say that Assoc. Prof. Hasanov is a thorough and serious scientist, teacher and leader. He is part of a very successful team of the University of Shumen working in the field of matrix and nonlinear equations.

### **8. Conclusion**

I firmly declare that the submitted materials in the competition fully comply with the requirements of ZRASRB and the Rules for implementation of ZRASRB.

In view of the above, I propose that Assoc. Prof. Dr. Vejdi Ismailov Hasanov be selected as "Professor" at the Faculty of Mathematics and Informatics of the Konstantin Preslavsky University of Shumen in the field of higher education 4. Natural sciences, mathematics and informatics, professional field 4.5 Mathematics (Computational Mathematics).

04.11.2019 г.

Gabrovo

Signature:



/Prof. A. Andreev/