Extended report

for the competition for the academic position "Professor" at Faculty of Mathematics and Computer Science, Konstantin Preslavsky University of Shumen announced in the State Gazette issue 63/06.08.2022, in the area of science 4. Natural sciences, mathematics and informatics, professional field 4.5 Mathematics, scientific direction *Probability theory and mathematical statistics* , prepared by Prof. Dr. Sc. Mladen Svetoslavov Savov, "Probability, Operations Research and Statistics", Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski" and "Operations research, Probability and Statistics", Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, appointed member of the scientific panel for the competition by the Chancellor of Konstantin Preslavsky University of Shumen, order No. 16.201/04.10.2022r., and writing an extended report according to the decision taken by the scientific panel during its first meeting on 11.10.2022.

This extended report was prepared in compliance with order No. 16.201/04.10.2022r. by the Chancellor of Konstantin Preslavsky University of Shumen issued on the basis of the decision by the Faculty Council of the Faculty of Mathematics and Computer Science (FMCS), Konstantin Preslavsky University of Shumen (Record No.FD-02-02/21.09.2022) in compliance with article 80, paragraphs 1 and 2 of the regulations for the Development of the Academic Staff of Konstantin Preslavsky University of Shumen, article 29a, paragraph 1 of the Act for the Development of the Academic Staff in the Republic of Bulgaria taken in relation to report filed by FMCS.

As a member of the scientific jury I have received all the documents submitted by the only applicant to the competition Associate Professor Pavlina Kalcheva Jordanova (Konstantin Preslavsky University of Shumen, FMCS).

1. Biographical data about the candidate

Pavlina Jordanova is born in 1972. In 1996 she graduated with a master degree in mathematics (specialization in econometrics) from FMCS of Konstantin Preslavsky University of Shumen and in 1998 she got a pedagogical qualification. In 2006 she obtained a PhD degree in *Probability theory and mathematical statistics* with a thesis titled "Multidimensional functional extremal criterion"and under the supervision of Prof. Dr.Sc. E. Pancheva. During the period from 1997 to 2014 Pavlina Jordanova was an assistant professor, a senior assistant professor and a chief assistant professor at Konstantin Preslavsky University of Shumen. In 2014 she was promoted to Associate Professor in *Probability theory and mathematical statistics* at Konstantin Preslavsky University of Shumen. Her main research interests include - extremal analysis, risk theory, probability theory, random processes, analysis of time series, financial mathematics.

2. Fulfilment of the minimal requirements

Assoc. Prof. Jordanova has furnished more than 45 citations and 21 papers for the competition: 3 out of the latter in Q1, 1 in Q2, 7 in Q3 and 1 in Q4. All minimal requirements in all categories are exceeded by a significant margin. All additional requirements are satisfied as well.

3. Research activity and scientific contributions of the candidate as testified by the applied documents

3.1. Overall evaluation of the scientific achievements of the candidate.

Assoc. Prof. Pavlina Jordanova has diverse and rich scientific work. This is complemented by her serious attitude towards science and I cannot refrain from mentioning that, in my view, one can see in her a successor of her PhD supervisor Prof. Dr. Sc. E. Pancheva. Assoc. Prof. Jordanova's results are along the following research lines: risk theory, statistical estimators, applications of probability to real world data, analysis of time-series, properties of stochastic processes and particular probability distributions. She has also mastered enough mathematical techniques from classical probability and she is capable of applying it in diverse contexts. I was particularly impressed by paper [13] because it reveals the ability of Assoc. Prof. Jordanova to find behind the numerous "multidimensional"risk models the classical and simple Cramer-Lundberg model. I must also note the intimate link of her research to applications highlighting the models related to data from Chile whence she has attempted to apply her theoretical results. On the critical side I ought to emphasize that some of her papers are unclearly written and one cannot easily understand the goals and the achievements therein. Also, there is too much focus on particular examples, which, in my view, at this stage of the career, must be replaced by more general ideas and results.

In the next part I will consider the achievements of the candidate according to their classification provided in the *Certificate for the original scientific contributions.*

3.2. Discussion of the particular scientific achievements of the candidate.

3.2.1. *A. Study of the probabilities for outside values (outliers) for different distributions - [monograph, 7, 8, 11].* I will discuss the content of Chapter 2 of the monograph, which is the pinnacle of this research line of Assoc. Prof. Jordanova who has results published in different venues. Chapter 2 puts the theoretical foundations for the introduction of statistics whose aim is to characterize the tail of a random variable X. In more detail, if $p \in (0, 1/2]$ and F is the cumulative distribution function of X, then the left- and the right- $p-$fences are defined as

$$L(X,p) = F^{\leftarrow}(p) - \frac{1-p}{p}\left(F^{\leftarrow}(1-p) - F^{\leftarrow}(p)\right);$$
$$R(X,p) = F^{\leftarrow}(1-p) + \frac{1-p}{p}\left(F^{\leftarrow}(1-p) - F^{\leftarrow}(p)\right).$$

Then the probabilities

$$p_{L,p}(X) := \mathbb{P}\left(X < L(X,p)\right), p_{R,p} := \mathbb{P}\left(X > R(X,p)\right),$$

are considered and they are the object of study of the monograph and are at the core of the developed methods for the determination of the class of distributions matching particular observations, and the subsequent specification of the precise distribution within the class above. The monograph contains the following properties of $p_{\cdot,p}(X)$: monotonicity in p, invariance with respect to linear transformations (their main advantage), calculation of $p_{\cdot,p}\left(g\left(X\right)\right)$ and inequalities, related to $p_{\cdot,p}\left(g\left(X\right)\right)$, particular formulae whenever $g(x) = a^x, g(x) = \log_a(x)$, $g(x) = x^{\alpha}$, evaluation of these probabilities for ordered statistics. The main properties of $p_{\cdot,p}(\cdot)$ demonstrate that they are independent both from translations and scaling (dilation) and from existence of moments. This is certainly an advantage when one studies the properties and the characteristics of the tail of distribution. The monograph presents particular computations for 22 distributions.

3.2.2. *B. Introduction of new statistics for extrema - [monograph, 1, 3, 4, 9, 19].* I shall exclusively comment chapters 3 and 4 of the monograph which collects most of the published results. Since in applied settings one works with data/observations from which one needs to estimate the parameters, let us say a vector α, of the random variable X, which is the prototype of the observations, the idea for the estimation of α is to employ the already introduced probabilities $p_{\cdot,p}(X)$ for $|\alpha|$ different values of p, and to solve at least one of the systems

$$p_{R,p_i}(X) = \hat{p}_R\left(p_i, n\right) \text{ or } \mathbb{P}\left(X > \hat{R}_n(p_i)\right) = \hat{p}_R\left(p_i, n\right); i \leq |\alpha|.$$

Above n is the number of observations and with $\hat{}$ we denote empirical quantities, which are based on the ordered statistics $X_{1:n}, \cdots, X_{n:n}$ of the data itself such as *the empirical quartile, the empirical proportion above the right fence, etc.* The solutions to these systems are called IPO-NM and IPO estimators respectively and the idea underpinning them resembles the method of moments for estimating the unknown parameters of certain distribution. In order to use these estimators one needs to check whether with the growth of the sample size the empirical quantities converge to the true parameters and in the limit sense the estimated ones coincide with the real. To obtain results in this direction the candidate has used the natural link between the distribution of the order statistics and the Beta random variable and she has obtained an expression for the density of $\hat{R}_{i+j-1}(i/(i+j))$. As a result, with some assumptions, it is shown that, for $s \geq 2$,

$$\lim_{i\to\infty} \mathbb{E}\left[\hat{R}_{(s+1)i-1}\left(\frac{1}{1+s}\right)\right] = R\left(X, \frac{1}{1+s}\right)$$
$$\lim_{i\to\infty} \hat{R}_{(s+1)i-1} \overset{\text{a.s.}}{=} R\left(X, \frac{1}{1+s}\right)$$
$$\lim_{i\to\infty} \sqrt{(s+1)i-1}\left(\hat{R}_{(s+1)i-1} - R\left(X, \frac{1}{1+s}\right)\right) \overset{d}{=} N(0,V).$$

For the proof of these results Assoc. Prof. Jordanova demonstrates excellent knowledge of limit theorems and mastering of complicated mathematical techniques.

When X is a particular random variable there are calculations which offer expressions not only for $p_{\cdot,p}(X), R(X,p)$ but also demonstrate how the estimators IPO-NM and IPO can be used for determining the unknown parameters of the law of X. When the systems are complicated then the candidate has employed numerical schemes for their solution, see [1,3].

The scientific contributions of Assoc. Prof. Jordanova include also a new approach for estimation of the parameter α of the Pareto, Fréchet, Log-logistic and Hill-horror distributions. This is achieved via a modification of the original approach of Hill in the following way: setting $\log(X_{(is,n)}/X_{(i,n)}) =: U_n(s,i)$, $s \geq 2$, the following quantities are considered[1]

$$Q^*_{i,s} := \frac{U_n(s,i)}{\log s};\ Q^{Fr*}_{i,s} := -\frac{U_n(s,i)}{\log\left(1 - \frac{\log s}{\log(s+1)}\right)};\ Q^{LL*}_{i,s} := \frac{1}{2}Q^*_{i,s};$$
$$Q^{HH*}_{i,s} := \frac{U_n(s,i) + \log\left(1 - \frac{\log s}{\log(s+1)}\right)}{\log s}$$

and, for $n = i(s+1)-1$, it is shown that

$$\lim_{i\to\infty} U_n(s,i) \overset{\text{a.s.}}{=} \log\left(\frac{F^{\leftarrow}_X(\frac{s}{s+1})}{F^{\leftarrow}_X(\frac{1}{s+1})}\right);\ \lim_{i\to\infty} \sqrt{i(s+1)-1}\left(U_n(s,i) - \log\left(\frac{F^{\leftarrow}_X(\frac{s}{s+1})}{F^{\leftarrow}_X(\frac{1}{s+1})}\right)\right) \overset{d}{=} N(0,V).$$

Given that we know $\log\left(\frac{F^{\leftarrow}_X(\frac{s}{s+1})}{F^{\leftarrow}_X(\frac{1}{s+1})}\right)$ which depends on the parameters of the law we can construct an estimator for these parameters without any additional knowledge about the moments and so on and so forth. This is done in the aforementioned cases and in the case of Pareto it is proved that, with $n = i(s+1)-1$, it holds that

$$\lim_{i\to\infty} Q^*_{i,s} \overset{\text{a.s.}}{=} \frac{1}{\alpha};\ \lim_{i\to\infty} Var(Q^*_{i,s}) = 0;\ \lim_{i\to\infty} \sqrt{i(s+1)-1}\,(\alpha Q^*_{i,s} - 1) \overset{d}{=} N(0,V).$$

[1] Since the two types of estimators are quite similar, I shall solely comment one of them.

This allows for the construction of asymptotic confidence intervals for α. There are also extensive numerical experiments. The results are dispersed in a few papers which are listed in the title of this section.

3.2.3. *C. Refinement of existing methods for time series (dynamical series) - [2, 12, 14, 17].* The contributions in this area are expected to be in the improvement of the methods for analysis of time series with focus on interest rates modelling. I have looked into the main papers on this topic and I have found them confusing. The main contributions are unclear with respect to how they conform to the main objectives. If I were a referee for some of them I would not have accepted them without a major revision. For example, paper [12] resembles a collection of facts, ideas and calculations, which vary from systems of stochastic differential equations, their discretization, the considerations of particular models and calculations related to them. Some of the theoretical results are a mere reformulation or a direct corollary of previous ones. Chapter 4 in [12] consists of computations (I do not get it why one quantity simultaneously stands for a number and for a random variable) without a specific aim. The same is more or less valid for the other papers. Therefore, it is almost impossible for me to comment on the candidate's achievements on this topic.

3.2.4. *D. Mathematical modelling of random processes in insurance mathematics and estimation of the probability for ruin of an insurance company - [10, 13, 16, 18, 20].* The applicant has a number of contributions to risk theory. Paper [13] demonstrates that a few multidimensional models can be reduced to the classical model of Cramer-Lundberg and using mainstream results for it the main quantities such as the ruin probability and the level of ruin can be computed as functions of the initial capital. From mathematical standpoint the results are not challenging, but this work was very much needed, since there had been a number of models with aspirations for novelty which in the end are clearly a special case of well-known theory[2]. Thus, a number of results by different authors, thanks to the paper of Assoc. Prof. Jordanova, are embedded into the classical ruin theory and in this way in the future there will be no need for their independent treatment. Paper [20] in its essence is a conditional renewal process based on the Exp-Pareto distribution of the step. In more detail, using one random variable $\Lambda_{\delta,\alpha}$[3] as mixing and considering its particular realization, the inter-arrival times have independent exponential times. Precisely, such a step is called Exp-Pareto. Then the conditional renewal process is mix Pareto-Poisson with respective parameters inherited from

[2]Regardless of how groups of claims are coming in, as long as the number follows Poisson counting process in time, then we have the classical model.

[3]We can think of $\Lambda_{\delta,\alpha}$ as a parameter whose particular chosen value in the beginning determines the behaviour of the system for the whole time horizon. This is something like the fundamental constants in physics, which are thought to be randomly chosen at the beginning of the Universe.

the Exp-Pareto law. For all involved random variables therein the applicant has considered their distributions, their properties and their main characterizing identities. Thus, for example, when the exponential step is considered, the sum of the inter-arrival times is the Erlang-Pareto law. The asymptotic properties of the renewal process are also considered and they depend on the particular value of $\Lambda_{\delta,\alpha}$. With any renewal process one can associate a risk process. This is also done in [20] and its main properties are studied. The paper has clear idea but heavy notations because of the involvement of more complicated quantities. Paper [16] has the same setting but with mixing random variable which randomizes the exponential step, i.e. the inter-arrival times ($Y_k \sim Exp(\Lambda)$). Practically speaking, the counting process conditioned on Λ is the classical Poisson process. Then its asymptotic behaviour is completely understood. In [16] a risk process, based on this renewal process, is introduced and an expression for the ruin probability on infinite horizon is yielded. Since the same probabilities on finite time interval are complicated for computation, an approximation of the risk process with a classical stochastic process is offered and then the probability for the ruin of the latter is considered. There are three cases. The first one is when the claims have a finite variance. The limit process is of course the classical Brownian motion and on this basis there are estimates for the ruin probability. However, they seem crude. It seems that the problem is in fact the problem for crossing $u_0 + \sqrt{t}$ by the Brownian motion and I think this case is one of the few studied in the literature. In the other two cases, i.e. when the variance is infinite, the limit process is stable. Then the problem for the approximation of the ruin probability is reduced to crossing by the stable process of a particular power function and it is possible that there are results in the theory for this as well. Paper [10] contains some computations for a random process generalizing the well-known Variance-Gamma process.

3.2.5. *E. Applications of the estimators and methods from the previous sections - [5, 6, 15].* The contributions of papers [5] and [6] are related to the application of the aforementioned estimators and methods to real data from Chile. It is interesting to note that the conclusion of [5], wherein temptingly one might conclude that the errors in a linear model are of a Pareto type but this is just a by-product of an outlier. This demonstrates deep knowledge of the applicant and her critical attitude towards results that can be interpreted in several different ways.

4. Teaching activity

Pavlina Jordanova has rich teaching experience and reading her CV I think that practically the teaching of all courses in probability and statistics at Konstantin Preslavsky University of Shumen is in one or another way related to herself during her career. For this reason she has basically taught all introductory courses in probability and statistics and some specialized ones. Assoc. Prof. Jordanova has also international teaching experience since she has been an Erasmus lector in Portugal, Austria and Poland.

5. Personal opinion about the candidate

I know Assoc. Prof. Jordanova since 2014. I can single out her serious attitude towards science and her high standards towards the results of her own papers. This, complemented by her mathematical abilities, makes Assoc. Prof. Jordanova an all-round scientist. I hope that in the future she will keep on her active research in probability and statistics.

6. Recommendation and overall evaluation

I would recommend the improvement of the candidate's academic writing and the decrease of the papers dealing with particular examples or special cases which have more or less known answer a priori. In addition I should like to suggest more active participation in the sessions of the National Stochastic Seminar which in turn may stimulate more scientific activities in our community.

7. Conclusion

According to the applied documents the only candidate Assoc. Prof. Pavlina Kalcheva Jordanova satisfies all the minimal requirements set by the Act for the Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation and the Regulations of Konstantin Preslavsky University of Shumen, stipulating the specific additional requirements for the award of scientific titles and academic positions. My personal opinion concerning her work is fully confirmed by the applied documents which clearly demonstrate that Assoc. Prof. Pavlina Jordanova is a very good specialist in the research area of the competition.

Therefore, I give an overall positive evaluation of Assoc. Prof. Pavlina Kalcheva Jordanova and I strongly recommend that the scientific panel approves of the candidate and proposes to the Faculty Council of FMCS of Konstantin Preslavsky University of Shumen to elect Assoc. Prof. Pavlina Kalcheva Jordanova as a Professor at Konstantin Preslavsky University of Shumen in professional field 4.5 Mathematics, scientific direction *Probability theory and mathematical statistics.*

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Prof. Dr.Sc. Mladen Savov

Sofia
15.11.2022