

REVIEW

by Prof. Dimcho Kostov Stankov, PhD

Concerning: Materials submitted for participation in a competition for academic position
"Associate Professor" of Bishop Konstantin Preslavsky University of Shumen
in Higher Education Area: 4. Natural sciences, mathematics and informatics
Professional Field: 4.5 Mathematics (Mathematical Analysis)

1. General presentation

For the participation in the announced competition for Associate Professor has submitted documents only one candidate assistant professor Aynur Abdulova Ali, PhD from the Department of Algebra and Geometry at the Faculty of Mathematics and Informatics of the Bishop Konstantin Preslavsky University of Shumen (ShU). The documents presented by her are in accordance with the Rules for the development of the academic staff of ShU.

In connection with the implementation of Art. 61 of the Regulations for the Implementation of the Law on the Development of the Academic Staff in the Republic of Bulgaria (LDASRB), from the submitted documents it was established that:

- a copy of diploma No. D - 233 of 12.05.2021 for the educational and scientific degree "doctor" in the implementation of Art. 60, para. 1;

- habilitation thesis - 2 scientific articles: 2 indexed in Web Of Science in quartile 1; equivalent to 150 points, with 100 points required;

- a report on the implementation of the minimum national requirements under Art. 2b, para. 2 and 3 of LDASRB, as well as a report on the original scientific contributions, to which the relevant evidence is attached, in the implementation of Art. 60, para. 3;

- a reference to the additional indicators applicable to the relevant field, in accordance with the requirements of Art. 61, para. 3;

- a declaration of authorship of the works in the competition (Art. 60, para. 1, item 6).

To participate in the competition, the candidate has proposed 5 scientific papers published in scientific journals. These publications do not repeat the ones submitted for the educational and scientific degree PhD. Assist. Prof. Aynur Ali has a total of 10 scientific papers and one dissertation for the acquisition of the degree "doctor".

II. Educational and pedagogical activity

Assistant Professor Ainur Ali graduated from the Faculty of Mathematics, OKS Bachelor in 2001. Since 2003 he has been a Master in Computational and Economic Mathematics at the University of Shumen. From 2013 to 2015 he has been a part-time lecturer. Since 2019 he has been a full-time assistant professor, and since 2022 he has been a chief assistant professor. In 2021 he defended his dissertation on the topic "Iterative methods for solving rational matrix equations".

Assis. Prof. Ainur Ali has lectured in Higher Mathematics, part two, for the specialty Engineering Logistics, part-time study at the Faculty of Technical Sciences /FTS/ and has led exercises as follows:

1. Mathematical Analysis Part I and Part II /Differential and Integral Calculus / for specialties from the Faculty of Mathematics, Faculty of Technology and the Faculty of Engineering;

2. Mathematical Analysis Part III and IV /Differential and Integral Calculus / for specialties from the Faculty of Mathematics and Physics;

3. Higher Mathematics Part I and II for specialties from the Faculty of Mathematics and Physics;

4. Mathematics and Applied Mathematics for specialties from the Faculty of Mathematics and Physics;

5. Elements of Set Theory, Mathematical Logic and Combinatorics for specialties from the Faculty of Mathematics and Physics.

6. Selected Chapters in Mathematics for the Master's Program in Economic Mathematics.

To help students, she prepares materials available on the university's e-learning platform. He methodologically supports the work of new entrants to the department.

III. Scientific and research activities.

Assist. Prof. A. Ali has submitted 5 publications in the competition. Two of the publications 4.1 and 4.2 are presented as an equalized equivalent number of articles under criterion B, and the rest under criterion D in accordance with the LDASRB. From the attached report on the fulfillment of the minimum national requirements under Art. 2b, para. 2 and 3 of LDASRB, it is clear that Assistant Professor A. Ali fulfills the minimum requirements for the groups of indicators for the academic position of Associate Professor in the professional field

4.5. Mathematics. She collects 481 points with a minimum number of 400 and meets the requirements for holding the academic position of associate professor.

The results in the submitted publications are in the field of fixed point theory and their applications in solving matrix equations and systems of matrix equations.

The articles submitted in the competition can be summarized in the following groups:

1. Applying the fixed-point pair theorem in a partially ordered metric space to find a positive definite solution to a system of matrix equations.

2. Study of triples of fixed points and their application in studying the dynamics of oligopolistic markets with three dominant firms and finding a positive definite solution to a system of matrix equations.

3. Application of the n -tupled fixed point theorem in a partially ordered metric space to find a positive definite solution to a system of matrix equations.

The presented articles are co-authored by mathematicians from Bulgaria.

The first group includes results in articles [A-1] and [A-2] from the list of publications in the reference for scientific contributions.

The problem of the existence of a pair of fixed points can be reduced to solving a system of symmetric equations:

In [9] B. Zlatanov developed the concept of pairs of fixed points so that arbitrary systems of two equations could be solved. Following the ideas in the [9] authors proved in [A-1] criteria for the existence and uniqueness of a solution to the system of matrices:

$$\begin{aligned} X &= Q - A^* X^{-1} A + B^* Y^{-1} B \\ Y &= R - C^* X^{-1} C + D^* Y^{-1} D \end{aligned}$$

where A , B , C and D are $N \times N$ Hermitian matrices, Q and R belong to $H(N)$ - the set of $N \times N$ Hermitian matrices with partial order and Q, R, A^*A, B^*B, C^*C и D^*D are positive definite matrices. This generalizes the result of A. Ran and M. Reurings on the [5] existence and uniqueness of fixed points in partially ordered metric spaces for monotone images, removing the requirement for continuity of the operator.

In [A-2] the authors consider the equation

$$X = Q + pA^* X^k A + qB^* X^l B$$

For $p, q \in \{\pm 1\}$ and $k, l \in [-1, 1] \setminus \{0\}$. Sufficient conditions for the existence and uniqueness of a solution have been found. The system has also been considered

$$X = Q + pA^* X^k A + qB^* Y^l B$$

$$Y = R + pC^* X^k C + qD^* Y^l D$$

where $k, l \in [-1, 1] \setminus \{0\}$, $p, q \in \{\pm 1\}$ and the above conditions for the participating matrices are satisfied. Sufficient conditions are proved for this system to have a unique solution. It is shown that the sign of p and the sign of q determine which partial ordering to choose in the Cartesian product $X \times X$.

The second group includes the results in the articles [A-3] and [A-4]. An oligopolistic market is a market with a small number of relatively large sellers. They dominate the market, but maintain competition among themselves. Therefore, an oligopolistic market combines competition and monopoly and is very often defined as “a market with competition between a few large ones”

In [A-3] statements about triples of fixed points are applied in the study of such markets. The authors study the dynamics of oligopolistic markets with three dominant firms using the reaction functions of the participants. They construct a model of the reaction of the participants. Analogously to the results in [A-1] and [A-2], results are obtained for the existence and uniqueness of fixed points for an ordered triple of images with (F, G, H) monotonicity of different types, with the partial ordering in the Cartesian product $X \times X$ arising from the images themselves.

In [A-4] sufficient conditions for the existence and uniqueness of a solution to a system of three nonlinear matrix equations are obtained:

$$X = Q_1 + p_1 A_1^* X^{-k_1} A_1 + p_2 B_1^* Y^{-k_2} B_1 + p_3 C_1^* Z^{-k_3} C_1$$

$$Y = Q_2 + p_1 A_2^* X^{-k_1} A_2 + p_2 B_2^* Y^{-k_2} B_2 + p_3 C_2^* Z^{-k_3} C_2,$$

$$Z = Q_3 + p_1 A_3^* X^{-k_1} A_3 + p_2 B_3^* Y^{-k_2} B_3 + p_3 C_3^* Z^{-k_3} C_3$$

where $p_i \in \{-1, 1\}$ and $k_i \in (0, 1]$ for $i = 1, 2, 3$. This generalizes the result of Ran and Reurings on [5] for existence and uniqueness of fixed points in partially ordered metric spaces for monotone mappings by removing the requirement for continuity of the operator.

The third group includes the results of the paper [A-5]. This paper uses techniques from [5], [6], [13], [17] and [18] for fixed points, pairs, triples, and N -triples of fixed points for images that have some kind of monotonicity for each of their variables. The authors weaken the classical requirement for contraction in partially ordered metric spaces by imposing the

condition that it is valid only for iteration series generated by the image. Again, the continuity requirement is replaced by a weaker condition.

Using the ideas of B. Samet and C. Vetro from [18] for N -torus fixed points of an image, it is generalized to N -torus images (F_1, F_2, \dots, F_N) . The result in [18] is obtained for a special choice of the images F_i , $i=1,2,\dots,N$. Sufficient conditions for the existence and uniqueness of solutions of various matrix equations are also proved. This result is illustrated by solving a particular matrix equation, which reduces to a symmetric system of matrix equations.

The results obtained are of serious scientific value. All articles in the competition have been published in journals with a high impact factor:

1. AIMS Mathematics – IF 1.8 ;
2. Mathematics – IF 2.4 ;
3. Axioms – IF 1.6.

At the time of submission of the documents, 11 citations have been noted. Senior Assistant Professor Ainur Ali is also a participant in research and educational projects.

1. At the university level – projects in 2020, 2022, 2023, 2024, 2025.
2. At the national level: National Program “Young Scientists and Postdoctoral Fellows – 2”, 2022-2025.

IV. Personal impressions and recommendations.

I know Assist. Prof. Ainur Ali as an established specialist in her field. I have no critical remarks about the scientific production submitted for participation in the competition. I recommend that she work with graduate students, and in years to come with doctoral students.

CONCLUSION

All of the above, as well as the fact that the candidate meets all the requirements of LDASRB, RALDASRB and the regulations of the University of Shumen, is the basis for my positive assessment and proposal to Assist. Prof. Aynur Ali to be awarded the academic position of "Associate Professor" in the field of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5 Mathematics (Mathematical Analysis).

Shumen
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Prepared the review:


(Prof. Dimcho Stankov)